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Towards New Teaching in Mathematics

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Tracking Errors – Children as Error Detectives

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Tracking Errors – Children as Error Detectives

> *"Errors are means to find the right path through a system, structure or network. We cannot learn if we are not allowed to make mistakes. The fear of making mistakes prevents us from discovering the unknown. We resort to automatisms without any insight and cognition. Committing mistakes in the learning environment should be a positive process, the starting point for further learning, motivation, and the quest and discovery of correlations."*

With this approach we transform subordination to self-responsibility, in other words, transfer some of the learning responsibility to the learners themselves. Interactive teaching and learning should also contribute to the teaching of mathematics. Every day we should create an environment that allows this approach and transforms passive imparting more and more to active demand, which strengthens the will to imbibe information and integrate it with the existing." (Jost and others, 1997, p. 34)

A challenge for planning of classroom teaching results from such an attitude to errors. Student errors are an important guide for the planning of teaching and targeted individual support. However, this is based on the prerequisite of creating a classroom environment where children can perceive errors as an opportunity for learning. A first consequence is to keep the importance of students' errors in lessons low. This is encouraged for example, by tasks that are not uniquely solvable, discussion of different computation methods, modelling tasks, development of new methods by students (e.g. discovery of methods for addition of fractions), self-initiated work as well as verbal or written arguments as a natural part of students' work. The more diverse such tasks are, the less the performance can be evaluated by the number of errors. Grounds for such teaching modules not only obviously result from positive perception of errors, but also from the focus on basic mathematics education and its quality criteria as a consequence from results of the Teaching-Learning Research (Klieme, 2003) and last but not least, from the increasing availability of calculators and computerised algebra systems. "If today our teaching is based on inculcating things into children that will be better done by calculating machines in a decade or two, we are calling for disaster." (Freudenthal, 1974)
In particular, if school classes deal with a subject through dialogue-based learning (Ruf / Gallin) (I do it this way, how will you do it, we shall do it so), the question of being right or wrong does not matter at first. Insofar dialogue-based learning can be compared with scientific theories: New theories emerge on the basis of previous observations and ideas and must be available for discussion and results of further experiments. Even a theory that is rejected may make an important contribution to further scientific development.

Errors in calculation result even in a class handling basic mathematics teaching. However, in an appropriately taught class, when children have become aware of the fact that the importance of these errors is secondary even for assessment, they can take up on them and increasingly assume personal responsibility for handling their mistakes. To allow students to think out loud, gives them and the teachers "indication of the relevant path and possible causes of error". "The object of error analysis is to find the reasons which have led to errors, and to draw out conclusions from them - both together with the child, if possible. The conclusions can affect the entire class in terms of preventive didactics.

At the same time they relate to the child as part of its personal background and development." (Jost and others, 1997, p. 33, 35) Error analysis is therefore a basis for individualised and support-focused instruction.

In particular, the teacher is questioned during the analysis in the elementary school. The increasing possibilities of error in the advanced curriculum of the secondary school level lead to the question of how the form of thinking aloud can be developed according to age. Even when children take over more responsibility, teachers should still have the chance to get an idea of the learning status of each child. These considerations led to the "Error-Assistant sheet" (template copy on page 8) as a tool for error detectives.

On work with the error-assistant sheet

While repeating the basic calculation methods in the 5th school year, in line with the method proposed by Margaret Schmassman (Jost and others, 1997, p. 43 f.) (see table), I presented at first two examples of false solutions:

Task:	Solution:
670 – 85	670 – 85 = 395
73 + = 100	73 + 72 = 100

Steps in the error analysis

Enter the error
Write down the assumptions
Asking the child
Highlighting the suspected error type
Thinking of ways to help

The challenge for detectives to find out what the person would have thought while solving or the reason for the error resulted in different assumptions.

However, most children gave the reason mentioned in Figure 1. In the subsequent period they were asked to enter errors in homework or in class in the left column and try to figure out why they made this mistake.
They could thereby be assisted by three classmates in their table group. I reminded about this ongoing task at regular intervals. Collecting the error-assistant sheets led to an overview of which children were successful as error detectives and with which ones frequent dialogues were held.
After the meeting, the children either accepted this example or entered their own explanation in their first error-assistant sheet.

Error	What I was thinking: What is wrong:	This is right:	Method of calculation					How often?
			+	−	·	:	S	
480 − 95 = 395	At first I subtracted 90 from 480 and then added 5 to the result	480 − 90 = 390 390 − 5 = 385		X				1
73 + 72 = 100	I received 27 as the result and reversed the numbers while writing it down.	73 + 27 = 100						

Figure 1: Error-assistant sheet (example).

The structure of the error-assistant sheet

The error-assistant sheet contains five columns. In the "Error" column the children enter the calculation that was detected as being wrong. If they were successful as error detectives, they record their thinking, or the error class in the second column. The right method is written in the third column. In principle, an explanation of the calculation method is desirable. The other two columns shall help to perceive clusters of errors in certain areas.
Not all children are able to recognise their own error patterns. They often write the text "I have calculated wrong" in the second column.

If the children want, they can present an error case in front of the class to use the skills of all detectives or in case of self-detected error patterns, to find out which children had the same thought and which other ways of thinking led to the same error. At the beginning, I always ask successful detectives to present their results. Experience shows that they are the students who learn better. When they present their errors, the threshold for under-performing students is also reduced.
The one, who found out the actual reason behind the mistake, has identified his own thinking pattern and thus, has the opportunity to change this. The presentation of various errors in the class also brings to light more and more types of errors, which can give other error detectives suggestions for their search.
The skills of the students while working with the error-assistant sheet can be subdivided roughly into three groups. Some students can analyse errors and explain the correct solution. Whereas due to lack of explanation of the other students, the extent of the correction on the basis of a new or reactivated understanding remains unclear. Statements like "I have miscalculated" in the third section shows that an independent individual work with the error-assistant sheet is still unfruitful. Besides the enthusiasm of the first group, support from the teacher is needed especially for the previous case. These children need brief support by thinking aloud with the teacher or in a small group.

A review of the error-assistant can be the basis for grouping in such a subsequent differentiation phase. Results of thinking aloud could be recorded together in the error-assistant sheet.

Repeated review of the error-assistant sheets shows learning progress to teachers and makes children realise their skills development through a feedback. Sample explanations from students and the joint work in a sub-group contribute to the promotion of speaking skills of students in the middle and the third group. Besides these tools for putting loud thinking in writing, individual tasks for the stabilisation of newly gained knowledge are useful, depending on the type of error. Control techniques or alternative computation methods should be trained, in particular for frequent concentration errors.
The error-assistant sheet shall support individual learning. Therefore, it should be used as a supplementary tool for teaching, homework or formative assessments. It is not suitable for correcting graded tests. Especially students, who were frustrated by a bad grade because of many miscalculations, could perceive learning progress through intensive work with the fault-assistant sheet.

In summary, the following points can be listed for working with error-assistant sheets (EA sheet):

- The EA sheet promotes personal responsibility in dealing with errors.
- It shows skills of students and thus, can be used as a tool for classification of differentiation groups.
- The ability to pen down loud thinking can be developed only over long period of time as many other skills.
- The EA sheet can increasingly reduce the teacher's workload, as more and more students work independently as error detectives.
- The EA sheet can contribute to positive perception of errors.
- The work with self-initiated thought patterns and reasons for proper solutions can enable or reinforce basic ideas.
- In the search for error causes with the entire class or a table group, it enhances interaction in the classroom.
- It can be seen that a calculation error may have different causes, and therefore the awareness of learning as an individual process is honed.
- EA sheets of a class give teachers the feedback to what extent educational objectives of previous lessons have been achieved and whether the basic ideas of students can be activated.

The idea for the error-assistant sheet was created during a course "Errors are always likely" (Ulla Kramer, Dominik Jost).

My Errors – my Assistants

Name: _____ Sheet No. _____

Error	What I was thinking: What is wrong:	This is right:	Method of calculation					How often?
			+	−	·	:	S	

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